

# YEAR 12 TO YEAR 13 TRANSITION BOOKLET

*Preparation for Y13 Further Mathematics*

## **Bridging the Gap from Year 12 to Year 13**

This booklet contains a structured collection of supplementary practice questions focusing on all CP1 content. Mastering these topics is essential for establishing fluency and maintaining a confident start to Year 13.



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## CP1 Transition Work

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### Core Pure Mathematics

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| Write your name here                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Surname                                                                                                                                   | Other names                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <b>Pearson</b><br><b>Edexcel GCE</b>                                                                                                      | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">           Centre Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> <div style="text-align: center;">           Candidate Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> </div> |
| <b>AS and A level Further Mathematics</b><br><b>Core Pure Mathematics</b><br><br><b>Practice Paper</b><br><b>Complex numbers (part 1)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables (Pink)                                                              | Total Marks<br><div style="border: 1px solid black; width: 60px; height: 40px; margin: 0 auto;"></div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

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- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 15 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
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### Advice

- Read each question carefully before you start to answer it.
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1.  $f(x) = 9x^3 - 33x^2 - 55x - 25.$

Given that  $x = 5$  is a solution of the equation  $f(x) = 0$ , use an algebraic method to solve  $f(x) = 0$  completely.

**(Total 5 marks)**

---

2.  $f(x) = 2x^3 - 6x^2 - 7x - 4.$

(a) Show that  $f(4) = 0$ .

**(1)**

(b) Use algebra to solve  $f(x) = 0$  completely.

**(4)**

**(Total 5 marks)**

---

3. The roots of the equation

$$2z^3 - 3z^2 + 8z + 5 = 0$$

are  $z_1$ ,  $z_2$  and  $z_3$ .

Given that  $z_1 = 1 + 2i$ , find  $z_2$  and  $z_3$ .

**(5)**

**(Total 5 marks)**

---

4. The complex numbers  $z_1$  and  $z_2$  are given by

$$z_1 = p + 2i \text{ and } z_2 = 1 - 2i$$

where  $p$  is an integer.

- (a) Find  $\frac{z_1}{z_2}$  in the form  $a + bi$  where  $a$  and  $b$  are real. Give your answer in its simplest form in terms of  $p$ .

(4)

Given that  $\left| \frac{z_1}{z_2} \right| = 13,$

- (b) find the possible values of  $p$ .

(4)

(Total 8 marks)

---

5. The complex numbers  $z$  and  $w$  are given by

$$z = 8 + 3i, \quad w = -2i$$

Express in the form  $a + bi$ , where  $a$  and  $b$  are real constants,

- (a)  $z - w$ ,

(1)

- (b)  $zw$ .

(2)

(Total 3 marks)

---

6. Given that  $z_1 = 1 - i$ ,

(a) find  $\arg(z_1)$ .

(2)

Given also that  $z_2 = 3 + 4i$ , find, in the form  $a + ib$ ,  $a, b \in \mathbb{R}$ ,

(b)  $z_1 z_2$ ,

(2)

(c)  $\frac{z_2}{z_1}$ .

(3)

In part (b) and part (c) you must show all your working clearly.

(Total 7 marks)

---

7.

$$z = 5 - 3i, \quad w = 2 + 2i$$

Express in the form  $a + bi$ , where  $a$  and  $b$  are real constants,

(a)  $z^2$ ,

(2)

(b)  $\frac{z}{w}$ .

(3)

(Total 5 marks)

---

8.  $z_1 = -2 + i$

(a) Find the modulus of  $z_1$ . (1)

(b) Find, in radians, the argument of  $z_1$ , giving your answer to 2 decimal places. (2)

The solutions to the quadratic equation

$$z^2 - 10z + 28 = 0$$

are  $z_2$  and  $z_3$ .

(c) Find  $z_2$  and  $z_3$ , giving your answers in the form  $p \pm i\sqrt{q}$ , where  $p$  and  $q$  are integers. (3)

(d) Show, on an Argand diagram, the points representing your complex numbers  $z_1$ ,  $z_2$  and  $z_3$ . (2)

(Total 8 marks)

---

9.  $z = \frac{50}{3 + 4i}$ .

Find, in the form  $a + ib$  where  $a, b \in \mathbb{R}$ ,

(a)  $z$ , (2)

(b)  $z^2$ . (2)

Find

(c)  $|z|$ , (2)

(d)  $\arg z^2$ , giving your answer in degrees to 1 decimal place. (2)

(Total 8 marks)

---

10. Given that 2 and  $1 - 5i$  are roots of the equation

$$x^3 + px^2 + 30x + q = 0, \quad p, q \in \mathbb{R}$$

- (a) write down the third root of the equation.

(1)

- (b) Find the value of  $p$  and the value of  $q$ .

(5)

- (c) Show the three roots of this equation on a single Argand diagram.

(2)

**Total 8 marks)**

---

11. Given that  $x = \frac{1}{2}$  is a root of the equation

$$2x^3 - 9x^2 + kx - 13 = 0, \quad k \in \mathbb{R}$$

find

- (a) the value of  $k$ ,

(3)

- (b) the other 2 roots of the equation.

(4)

**(Total 7 marks)**

---



12. (i) The complex number  $w$  is given by

$$w = \frac{p - 4i}{2 - 3i}$$

where  $p$  is a real constant.

- (a) Express  $w$  in the form  $a + bi$ , where  $a$  and  $b$  are real constants.  
Give your answer in its simplest form in terms of  $p$ .

(3)

Given that  $\arg w = \frac{\pi}{4}$

- (b) find the value of  $p$ .

(2)

- (ii) The complex number  $z$  is given by

$$z = (1 - \lambda i)(4 + 3i)$$

where  $\lambda$  is a real constant.

Given that

$$|z| = 45$$

find the possible values of  $\lambda$ .

Give your answers as exact values in their simplest form.

(3)

(Total 8 marks)

---

13.

$$z_1 = 3i \text{ and } z_2 = \frac{6}{1 + i\sqrt{3}}.$$

- (a) Express  $z_2$  in the form  $a + ib$ , where  $a$  and  $b$  are real numbers.

(2)

- (b) Find the modulus and the argument of  $z_2$ , giving the argument in radians in terms of  $\pi$ .

(4)

- (c) Show the three points representing  $z_1$ ,  $z_2$  and  $(z_1 + z_2)$  respectively, on a single Argand diagram.

(2)

(Total 8 marks)

---

14. The complex number  $z$  is given by

$$z = \frac{p+2i}{3+pi}$$

where  $p$  is an integer.

- (a) Express  $z$  in the form  $a + bi$  where  $a$  and  $b$  are real. Give your answer in its simplest form in terms of  $p$ .

(4)

- (b) Given that  $\arg(z) = \theta$ , where  $\tan \theta = 1$  find the possible values of  $p$ .

(5)

(Total 9 marks)

---

15.  $f(x) = (4x^2 + 9)(x^2 - 2x + 5)$

- (a) Find the four roots of  $f(x) = 0$ .

(4)

- (b) Show the four roots of  $f(x) = 0$  on a single Argand diagram.

(2)

(Total 6 marks)

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**TOTAL FOR PAPER: 100 MARKS**

|                                                                                                                                           |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
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- Try to answer every question.
- Check your answers if you have time at the end.
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1.

$$z = \frac{4}{1+i}.$$

Find, in the form  $a + ib$  where  $a, b \in \mathbb{R}$ ,

(a)  $z$ , (2)

(b)  $z^2$ . (2)

Given that  $z$  is a complex root of the quadratic equation  $x^2 + px + q = 0$ , where  $p$  and  $q$  are real integers,

(c) find the value of  $p$  and the value of  $q$ . (3)

(Total 7 marks)

---

2.

$$f(x) = (4x^2 + 9)(x^2 - 6x + 34).$$

(a) Find the four roots of  $f(x) = 0$ .

Give your answers in the form  $x = p + iq$ , where  $p$  and  $q$  are real.

(5)

(b) Show these four roots on a single Argand diagram.

(2)

(Total 7 marks)

---

3. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are  $z_1$ ,  $z_2$  and  $z_3$ .

- (a) Given that  $z_1 = 3 + i$ , find  $z_2$  and  $z_3$ .

(4)

- (b) Show, on a single Argand diagram, the points representing  $z_1$ ,  $z_2$  and  $z_3$ .

(2)

(Total 6 marks)

---

4. Given that 4 and  $2i - 3$  are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where  $a$  and  $b$  are real constants,

- (a) write down the third root of the equation,

(1)

- (b) find the value of  $a$  and the value of  $b$ .

(5)

(Total 6 marks)

---

5. Given that  $z = x + iy$ , find the value of  $x$  and the value of  $y$  such that

$$z + 3iz^* = -1 + 13i$$

where  $z^*$  is the complex conjugate of  $z$ .

(Total 7 marks)

---

6. A complex number  $z$  is given by  $z = a + 2i$ ,

where  $a$  is a non-zero real number.

- (a) Find  $z^2 + 2z$  in the form  $x + iy$  where  $x$  and  $y$  are real expressions in terms of  $a$ . (4)

Given that  $z^2 + 2z$  is real,

- (b) find the value of  $a$ . (1)

Using this value for  $a$ ,

- (c) find the values of the modulus and argument of  $z$ , giving the argument in radians, and giving your answers to 3 significant figures. (3)

- (d) Show the points  $P$ ,  $Q$  and  $R$ , representing the complex numbers  $z$ ,  $z^2$  and  $z^2 + 2z$  respectively, on a single Argand diagram with origin  $O$ . (3)

- (e) Describe fully the geometrical relationship between the line segments  $OP$  and  $QR$ . (2)

(Total 13 marks)

---

7.  $z_1 = 2 + 3i$ ,  $z_2 = 3 + 2i$ ,  $z_3 = a + bi$ ,  $a, b \in \mathbb{R}$

- (a) Find the exact value of  $|z_1 + z_2|$ . (2)

Given that  $w = \frac{z_1 z_3}{z_2}$ ,

- (b) find  $w$  in terms of  $a$  and  $b$ , giving your answer in the form  $x + iy$ ,  $x, y \in \mathbb{R}$  (4)

Given also that  $w = \frac{17}{13} - \frac{7}{13}i$ ,

- (c) find the value of  $a$  and the value of  $b$ , (3)

- (d) find  $\arg w$ , giving your answer in radians to 3 decimal places. (2)

(Total 11 marks)

---

8.  $z = 2 - i\sqrt{3}.$

(a) Calculate  $\arg z$ , giving your answer in radians to 2 decimal places.

(2)

Use algebra to express

(b)  $z + z^2$  in the form  $a + bi\sqrt{3}$ , where  $a$  and  $b$  are integers,

(3)

(c)  $\frac{z+7}{z-1}$  in the form  $c + di\sqrt{3}$ , where  $c$  and  $d$  are integers.

(4)

Given that  $w = \lambda - 3i$ ,

where  $\lambda$  is a real constant, and  $\arg(4 - 5i + 3w) = -\frac{\pi}{2}$ ,

(d) find the value of  $\lambda$ .

(2)

**(Total 11 marks)**

---

9.  $z = -24 - 7i$

(a) Show  $z$  on an Argand diagram.

(1)

(b) Calculate  $\arg z$ , giving your answer in radians to 2 decimal places.

(2)

It is given that  $w = a + bi$ ,  $a \in \mathbb{R}$ ,  $b \in \mathbb{R}$ .

Given also that  $|w| = 4$  and  $\arg w = \frac{5\pi}{6}$ ,

(c) find the values of  $a$  and  $b$ ,

(3)

(d) find the value of  $|zw|$ .

(3)

**(Total 9 marks)**

---

10. The point  $P$  represents a complex number  $z$  on an Argand diagram such that

$$|z - 6i| = 2|z - 3|.$$

- (a) Show that, as  $z$  varies, the locus of  $P$  is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point  $Q$  represents a complex number  $z$  on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}.$$

- (b) Sketch, on the same Argand diagram, the locus of  $P$  and the locus of  $Q$  as  $z$  varies.

(4)

- (c) Find the complex number for which both  $|z - 6i| = 2|z - 3|$  and  $\arg(z - 6) = -\frac{3\pi}{4}$ .

(4)

**(Total 14 marks)**

---



11. The complex number  $w$  is given by

$$w = 10 - 5i$$

- (a) Find  $|w|$ .

(1)

- (b) Find  $\arg w$ , giving your answer in radians to 2 decimal places

(2)

The complex numbers  $z$  and  $w$  satisfy the equation

$$(2 + i)(z + 3i) = w$$

- (c) Use algebra to find  $z$ , giving your answer in the form  $a + bi$ ,  
where  $a$  and  $b$  are real numbers.

(4)

Given that

$$\arg(\lambda + 9i + w) = \frac{\pi}{4}$$

where  $\lambda$  is a real constant,

- (d) find the value of  $\lambda$ .

(2)

(Total 9 marks)

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**TOTAL FOR PAPER: 100 MARKS**

|                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
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|                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                   |
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1.

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix  $\mathbf{M}$  is singular, find the possible values of  $x$ .

**(Total 4 marks)**

---

2.

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \quad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(a) Find  $\mathbf{A}^{-1}$

**(2)**

The transformation represented by the matrix  $\mathbf{B}$  followed by the transformation represented by the matrix  $\mathbf{A}$  is equivalent to the transformation represented by the matrix  $\mathbf{P}$ .

(b) Find  $\mathbf{B}$ , giving your answer in its simplest form.

**(3)**

**(Total 5 marks)**

---

3. (i)  $\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}$ , where  $k$  is a constant

Given that  $\mathbf{B} = \mathbf{A} + 3\mathbf{I}$

where  $\mathbf{I}$  is the  $2 \times 2$  identity matrix, find

(a)  $\mathbf{B}$  in terms of  $k$ ,

(2)

(b) the value of  $k$  for which  $\mathbf{B}$  is singular.

(2)

(ii) Given that  $\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}$ ,  $\mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$

and

$$\mathbf{E} = \mathbf{CD}$$

find  $\mathbf{E}$ .

(2)

(Total 6 marks)

---

4. (a) Given that  $\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix}$ ,

find  $\mathbf{AB}$ .

(2)

(b) Given that  $\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix}$  and  $\mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}$ , where  $k$  is a constant

and

$$\mathbf{E} = \mathbf{C} + \mathbf{D},$$

find the value of  $k$  for which  $\mathbf{E}$  has no inverse.

(4)

(Total 6 marks)

---

5.

$$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(a) Find  $\mathbf{AB}$ .

(3)

Given that

$$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(b) describe fully the geometrical transformation represented by  $\mathbf{C}$ ,

(2)

(c) write down  $\mathbf{C}^{100}$ .

(1)

(Total 6 marks)

6. (a) Given that

$$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix},$$

(i) find  $\mathbf{A}^2$ ,

(ii) describe fully the geometrical transformation represented by  $\mathbf{A}^2$ .

(4)

(b) Given that

$$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

describe fully the geometrical transformation represented by  $\mathbf{B}$ .

(2)

(c) Given that

$$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix},$$

where  $k$  is a constant, find the value of  $k$  for which the matrix  $\mathbf{C}$  is singular.

(3)

(Total 9 marks)

7. (i) Given that  $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$  and  $\mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$ ,

(a) find  $\mathbf{AB}$ .

(b) Explain why  $\mathbf{AB} \neq \mathbf{BA}$ .

(4)

(ii) Given that  $\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}$ , where  $k$  is a real number

find  $\mathbf{C}^{-1}$ , giving your answer in terms of  $k$ .

(3)

(Total 7 marks)

8. The transformation  $U$ , represented by the  $2 \times 2$  matrix  $\mathbf{P}$ , is a rotation through  $90^\circ$  anticlockwise about the origin.

(a) Write down the matrix  $\mathbf{P}$ .

(1)

The transformation  $V$ , represented by the  $2 \times 2$  matrix  $\mathbf{Q}$ , is a reflection in the line  $y = -x$ .

(b) Write down the matrix  $\mathbf{Q}$ .

(1)

Given that  $U$  followed by  $V$  is transformation  $T$ , which is represented by the matrix  $\mathbf{R}$ ,

(c) express  $\mathbf{R}$  in terms of  $\mathbf{P}$  and  $\mathbf{Q}$ ,

(1)

(d) find the matrix  $\mathbf{R}$ ,

(2)

(e) give a full geometrical description of  $T$  as a single transformation.

(2)

(Total 7 marks)

9. A right angled triangle  $T$  has vertices  $A(1, 1)$ ,  $B(2, 1)$  and  $C(2, 4)$ . When  $T$  is transformed by the matrix  $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ , the image is  $T'$ .

(a) Find the coordinates of the vertices of  $T'$ .

(2)

(b) Describe fully the transformation represented by  $\mathbf{P}$ .

(2)

The matrices  $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$  and  $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$  represent two transformations. When  $T$  is transformed by the matrix  $\mathbf{QR}$ , the image is  $T''$ .

(c) Find  $\mathbf{QR}$ .

(2)

(d) Find the determinant of  $\mathbf{QR}$ .

(2)

(e) Using your answer to part (d), find the area of  $T''$ .

(3)

(Total 11 marks)

---

10. (i)

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$$

where  $p$  is a constant.

(a) Find, in terms of  $p$ , the matrix  $\mathbf{AB}$

(2)

Given that

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

where  $k$  is a constant and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix,

(b) find the value of  $p$  and the value of  $k$ .

(4)

(ii)

$$\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}, \text{ where } a \text{ is a real constant}$$

Triangle  $T$  has an area of 15 square units.

Triangle  $T$  is transformed to the triangle  $T'$  by the transformation represented by the matrix  $\mathbf{M}$ .

Given that the area of triangle  $T'$  is 270 square units, find the possible values of  $a$ .

(5)

(Total 11 marks)

---

11.

$$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix  $\mathbf{A}$  maps the point with coordinates  $(4, 6)$  onto the point with coordinates  $(2, -8)$ ,

(a) find the value of  $a$  and the value of  $b$ .

(4)

A quadrilateral  $R$  has area 30 square units.

It is transformed into another quadrilateral  $S$  by the matrix  $\mathbf{A}$ .

Using your values of  $a$  and  $b$ ,

(b) find the area of quadrilateral  $S$ .

(4)

(Total 8 marks)



12.

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

- (a) Describe fully the single geometrical transformation  $U$  represented by the matrix  $\mathbf{P}$ . (2)

The transformation  $U$  maps the point  $A$ , with coordinates  $(p, q)$ , onto the point  $B$ , with coordinates  $(6\sqrt{2}, 3\sqrt{2})$ .

- (b) Find the value of  $p$  and the value of  $q$ . (3)

The transformation  $V$ , represented by the  $2 \times 2$  matrix  $\mathbf{Q}$ , is a reflection in the line with equation  $y = x$ .

- (c) Write down the matrix  $\mathbf{Q}$ . (1)

The transformation  $U$  followed by the transformation  $V$  is the transformation  $T$ . The transformation  $T$  is represented by the matrix  $\mathbf{R}$ .

- (d) Find the matrix  $\mathbf{R}$ . (3)

- (e) Deduce that the transformation  $T$  is self-inverse. (1)

(Total 10 marks)

13.

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that  $\mathbf{M} = (\mathbf{A} + \mathbf{B})(2\mathbf{A} - \mathbf{B})$ ,

- (a) calculate the matrix  $\mathbf{M}$ , (6)

- (b) find the matrix  $\mathbf{C}$  such that  $\mathbf{MC} = \mathbf{A}$ . (4)

(Total 10 marks)

**TOTAL FOR PAPER: 100 MARKS**

|                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| Write your name here                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Surname                                                                                                                                  | Other names                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <b>Pearson</b><br><b>Edexcel GCE</b>                                                                                                     | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">           Centre Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> <div style="text-align: center;">           Candidate Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> </div> |
| <b>AS and A level Further Mathematics</b><br><b>Core Pure Mathematics</b><br><br><b>Practice Paper</b><br><b>Matrix algebra (part 2)</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables (Pink)                                                             | <div style="border: 1px solid black; width: 80px; height: 40px; margin: 0 auto;">Total Marks</div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

### Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 9 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a \* sign.

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i)

$$\mathbf{A} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \end{pmatrix}$$

(a) Describe fully the single transformation represented by the matrix  $\mathbf{A}$ .

(2)

The matrix  $\mathbf{B}$  represents an enlargement, scale factor  $-2$ , with centre the origin.

(b) Write down the matrix  $\mathbf{B}$ .

(1)

(ii)

$$\mathbf{M} = \begin{pmatrix} 3 & k \\ -2 & 3 \end{pmatrix}, \quad \text{where } k \text{ is a positive constant.}$$

Triangle  $T$  has an area of 16 square units.

Triangle  $T$  is transformed onto the triangle  $T'$  by the transformation represented by the matrix  $\mathbf{M}$ .

Given that the area of the triangle  $T'$  is 224 square units, find the value of  $k$ .

(3)

(Total 6 marks)

---

2.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$$

The transformation represented by  $\mathbf{B}$  followed by the transformation represented by  $\mathbf{A}$  is equivalent to the transformation represented by  $\mathbf{P}$ .

(a) Find the matrix  $\mathbf{P}$ .

(2)

Triangle  $T$  is transformed to the triangle  $T'$  by the transformation represented by  $\mathbf{P}$ .

Given that the area of triangle  $T'$  is 24 square units,

(b) find the area of triangle  $T$ .

(3)

Triangle  $T'$  is transformed to the original triangle  $T$  by the matrix represented by  $\mathbf{Q}$ .

(c) Find the matrix  $\mathbf{Q}$ .

(2)

(Total 7 marks)

3.

$$\mathbf{X} = \begin{pmatrix} 1 & a \\ 3 & 2 \end{pmatrix}, \text{ where } a \text{ is a constant.}$$

(a) Find the value of  $a$  for which the matrix  $\mathbf{X}$  is singular.

(2)

$$\mathbf{Y} = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}.$$

(b) Find  $\mathbf{Y}^{-1}$ .

(2)

The transformation represented by  $\mathbf{Y}$  maps the point  $A$  onto the point  $B$ .

Given that  $B$  has coordinates  $(1 - \lambda, 7\lambda - 2)$ , where  $\lambda$  is a constant,

(c) find, in terms of  $\lambda$ , the coordinates of point  $A$ .

(4)

(Total 8 marks)

4. (i)  $\mathbf{A} = \begin{pmatrix} 5k & 3k-1 \\ -3 & k+1 \end{pmatrix}$ , where  $k$  is a real constant.

Given that  $\mathbf{A}$  is a singular matrix, find the possible values of  $k$ .

(4)

(ii)  $\mathbf{B} = \begin{pmatrix} 10 & 5 \\ -3 & 3 \end{pmatrix}$

A triangle  $T$  is transformed onto a triangle  $T'$  by the transformation represented by the matrix  $\mathbf{B}$ .

The vertices of triangle  $T'$  have coordinates  $(0, 0)$ ,  $(-20, 6)$  and  $(10c, 6c)$ , where  $c$  is a positive constant.

The area of triangle  $T'$  is 135 square units.

(a) Find the matrix  $\mathbf{B}^{-1}$ .

(2)

(b) Find the coordinates of the vertices of the triangle  $T$ , in terms of  $c$  where necessary.

(3)

(c) Find the value of  $c$ .

(3)

(Total 12 marks)

5. (i) In each of the following cases, find a  $2 \times 2$  matrix that represents

(a) a reflection in the line  $y = -x$ ,

(b) a rotation of  $135^\circ$  anticlockwise about  $(0, 0)$ ,

(c) a reflection in the line  $y = -x$  followed by a rotation of  $135^\circ$  anticlockwise about  $(0, 0)$ .

(4)

(ii) The triangle  $T$  has vertices at the points  $(1, k)$ ,  $(3, 0)$  and  $(11, 0)$ , where  $k$  is a constant. Triangle  $T$  is transformed onto the triangle  $T'$  by the matrix

$$\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix}$$

Given that the area of triangle  $T'$  is 364 square units, find the value of  $k$ .

(6)

(Total 10 marks)

6.

$$\mathbf{A} = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}$$

and  $\mathbf{I}$  is the  $2 \times 2$  identity matrix.

(a) Prove that

$$\mathbf{A}^2 = 7\mathbf{A} + 2\mathbf{I} \quad (2)$$

(b) Hence show that

$$\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A} - 7\mathbf{I}) \quad (2)$$

The transformation represented by  $\mathbf{A}$  maps the point  $P$  onto the point  $Q$ .

Given that  $Q$  has coordinates  $(2k + 8, -2k - 5)$ , where  $k$  is a constant,

(c) find, in terms of  $k$ , the coordinates of  $P$ . (4)

(Total 8 marks)

---

7.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ 2 & 3 \end{pmatrix}.$$

(a) Show that  $\mathbf{A}$  is non-singular. (2)

(b) Find  $\mathbf{B}$  such that  $\mathbf{BA}^2 = \mathbf{A}$ . (4)

(Total 6 marks)

---

8.

$$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$$

(a) Find  $\det \mathbf{A}$ .

(1)

(b) Find  $\mathbf{A}^{-1}$ .

(2)

The triangle  $R$  is transformed to the triangle  $S$  by the matrix  $\mathbf{A}$ .

Given that the area of triangle  $S$  is 72 square units,

(c) find the area of triangle  $R$ .

(2)

The triangle  $S$  has vertices at the points  $(0, 4)$ ,  $(8, 16)$  and  $(12, 4)$ .

(d) Find the coordinates of the vertices of  $R$ .

(4)

**(Total 9 marks)**

---

9.

$$\mathbf{M} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}.$$

(a) Find  $\det \mathbf{M}$ .

(1)

The transformation represented by  $\mathbf{M}$  maps the point  $S(2a - 7, a - 1)$ , where  $a$  is a constant, onto the point  $S'(25, -14)$ .

(b) Find the value of  $a$ .

(3)

The point  $R$  has coordinates  $(6, 0)$ .

Given that  $O$  is the origin,

(c) find the area of triangle  $ORS$ .

(2)

Triangle  $ORS$  is mapped onto triangle  $OR'S'$  by the transformation represented by  $\mathbf{M}$ .

(d) Find the area of triangle  $OR'S'$ .

(2)

Given that

$$\mathbf{A} = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(e) describe fully the single geometrical transformation represented by  $\mathbf{A}$ .

(2)

The transformation represented by  $\mathbf{A}$  followed by the transformation represented by  $\mathbf{B}$  is equivalent to the transformation represented by  $\mathbf{M}$ .

(f) Find  $\mathbf{B}$ .

(4)

(Total 14 marks)

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**TOTAL FOR PAPER: 100 MARKS**



|                                                                                                                        |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| <b>Pearson</b><br><b>Edexcel GCE</b>                                                                                   | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">           Centre Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> <div style="text-align: center;">           Candidate Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> </div> |
| <b>AS and A level Further Mathematics</b><br><b>Core Pure Mathematics</b><br><br><b>Practice Paper</b><br><b>Proof</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables (Pink)                                           | Total Marks<br><div style="border: 1px solid black; width: 80px; height: 40px; margin: 0 auto;"></div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                    |

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- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 6 questions in this question paper. The total mark for this paper is 53.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a \* sign.

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. (i) Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^n - 1) & 5^n \end{pmatrix}$$

(6)

- (ii) Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(4n^2 - 1)$$

(6)

**(Total 12 marks)**

---

2. (i) A sequence of numbers is defined by

$$\begin{aligned} u_1 &= 6, & u_2 &= 27 \\ u_{n+2} &= 6u_{n+1} - 9u_n & n &\geq 1 \end{aligned}$$

Prove by induction that, for  $n \in \mathbb{Z}^+$

$$u_n = 3^n(n+1)$$

(6)

- (ii) Prove by induction that, for  $n \in \mathbb{Z}^+$

$$f(n) = 3^{3n-2} + 2^{3n+1} \text{ is divisible by 19}$$

(6)

**(Total 12 marks)**

---

3. Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

$$f(n) = 8^n - 2^n$$

is divisible by 6.

**(Total 6 marks)**

---

4. Prove by induction, that for  $n \in \mathbb{Z}^+$ ,

$$(a) \begin{pmatrix} 3 & 0 \\ 6 & 1 \end{pmatrix}^n = \begin{pmatrix} 3^n & 0 \\ 3(3^n - 1) & 1 \end{pmatrix} \quad (6)$$

$$(b) f(n) = 7^{2n-1} + 5 \text{ is divisible by } 12. \quad (6)$$

**(Total 12 marks)**

---

5. A sequence of numbers  $u_1, u_2, u_3, u_4, \dots$ , is defined by

$$u_{n+1} = 4u_n + 2, \quad u_1 = 2.$$

Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

$$u_n = \frac{2}{3}(4^n - 1). \quad (5)$$

**(Total 5 marks)**

---

6. Prove by induction that, for  $n \in \mathbb{Z}^+$ ,

$$f(n) = 2^{2n-1} + 3^{2n-1}$$

is divisible by 5.

**(Total 6 marks)**

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**TOTAL FOR PAPER: 53 MARKS**

|                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| Surname                                                                                                                 | Other names                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <b>Pearson</b><br><b>Edexcel GCE</b>                                                                                    | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">           Centre Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> <div style="text-align: center;">           Candidate Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> </div> |
| <b>AS and A level Further Mathematics</b><br><b>Core Pure Mathematics</b><br><br><b>Practice Paper</b><br><b>Series</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| <b>You must have:</b><br>Mathematical Formulae and Statistical Tables (Pink)                                            | <div style="border: 1px solid black; width: 80px; height: 40px; margin: 0 auto;">Total Marks</div>                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                        |

### Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a \* sign.

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Show, using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ , that

$$\sum_{r=1}^n 3(2r-1)^2 = n(2n+1)(2n-1), \text{ for all positive integers } n.$$

(Total 5 marks)

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2. (a) Using the formula for  $\sum_{r=1}^n r^2$  write down, in terms of  $n$  only, an expression for

$$\sum_{r=1}^{3n} r^2$$

(1)

- (b) Show that, for all integers  $n$ , where  $n > 0$ ,

$$\sum_{r=2n+1}^{3n} r^2 = \frac{n}{6}(an^2 + bn + c)$$

where the values of the constants  $a$ ,  $b$  and  $c$  are to be found.

(4)

(Total 5 marks)

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3. (a) Using the formulae for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$ , show that

$$\sum_{r=1}^n (r+1)(r+4) = \frac{n}{3}(n+4)(n+5)$$

for all positive integers  $n$ .

(5)

- (b) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{n}{3}(n+1)(an+b)$$

where  $a$  and  $b$  are integers to be found.

(3)

(Total 8 marks)

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4. (a) Use the standard results for  $\sum_{r=1}^n r^3$  and  $\sum_{r=1}^n r$  to show that

$$\sum_{r=1}^n (r^3 + 6r - 3) = \frac{1}{4}n^2(n + 2n + 13)$$

for all positive integers  $n$ .

(5)

- (b) Hence find the exact value of

$$\sum_{r=16}^{30} (r^3 + 6r - 3).$$

(2)

(Total 7 marks)

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5. (a) Use the results for  $\sum_{r=1}^n r$ ,  $\sum_{r=1}^n r^2$  and  $\sum_{r=1}^n r^3$ , to prove that

$$\sum_{r=1}^n r(r+1)(r+5) = \frac{1}{4}n(n+1)(n+2)(n+7)$$

for all positive integers  $n$ .

(5)

- (b) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5)$$

(2)

(Total 7 marks)

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6. (a) Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^3$  to show that

$$\sum_{r=1}^n r(r^2 - 3) = \frac{1}{4}n(n+1)(n+3)(n-2)$$

(5)

- (b) Calculate the value of  $\sum_{r=10}^{50} r(r^2 - 3)$ .

(3)

(Total 8 marks)

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7. (a) Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$  to show that

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(4n^2 - 1)$$

(6)

- (b) Hence show that

$$\sum_{r=2n+1}^{4n} (2r-1)^2 = an(bn^2 - 1)$$

where  $a$  and  $b$  are constants to be found.

(3)

(Total 9 marks)

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8. (a) Use the standard results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$  to show that

$$\sum_{r=1}^n (r+2)(r+3) = \frac{1}{3}n(n^2 + 9n + 26)$$

for all positive integers  $n$ .

(6)

- (b) Hence show that

$$\sum_{r=n+1}^{3n} (r+2)(r+3) = \frac{2}{3}n(an^2 + bn + c)$$

where  $a$ ,  $b$  and  $c$  are integers to be found.

(4)

(Total 10 marks)

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9. (a) Use the results for  $\sum_{r=1}^n r$  and  $\sum_{r=1}^n r^2$  to show that

$$\sum_{r=1}^n (2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers  $n$ .

(6)

- (b) Hence show that

$$\sum_{r=n+1}^{3n} (2r-1)^2 = \frac{2}{3}n(an^2 + b)$$

where  $a$  and  $b$  are integers to be found.

(4)

(Total 10 marks)

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10. (a) Prove by induction

$$\sum_{r=1}^n r^3 = \frac{1}{4}n^2(n+1)^2.$$

(5)

(b) Using the result in part (a), show that

$$\sum_{r=1}^n (r^3 - 2) = \frac{1}{4}n(n^3 + 2n^2 + n - 8).$$

(3)

(c) Calculate the exact value of  $\sum_{r=20}^{50} (r^3 - 2)$ .

(3)

(Total 11 marks)

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**TOTAL FOR PAPER: 80 MARKS**

|                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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| Write your name here                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
| Surname                                                                                                                  | Other names                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |
| <b>Pearson</b><br><b>Edexcel GCE</b>                                                                                     | <div style="display: flex; justify-content: space-around;"> <div style="text-align: center;">           Centre Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> <div style="text-align: center;">           Candidate Number<br/> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> <div style="border: 1px solid black; width: 30px; height: 30px; margin: 0 auto;"></div> </div> </div> |
| <b>AS and A level Further Mathematics</b><br><b>Core Pure Mathematics</b><br><br><b>Practice Paper</b><br><b>Vectors</b> |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
|                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                           |
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- Inexact answers should be given to three significant figures unless otherwise stated.

### Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 8 questions in this question paper. The total mark for this paper is 101.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a \* sign.

### Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. With respect to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 5 \\ -3 \\ p \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ 1 \\ -3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} 8 \\ 5 \\ -2 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 4 \\ -5 \end{pmatrix},$$

where  $\lambda$  and  $\mu$  are scalar parameters and  $p$  is a constant.

The lines  $l_1$  and  $l_2$  intersect at the point  $A$ .

- (a) Find the coordinates of  $A$ .

(2)

- (b) Find the value of the constant  $p$ .

(3)

- (c) Find the acute angle between  $l_1$  and  $l_2$ , giving your answer in degrees to 2 decimal places.

(3)

The point  $B$  lies on  $l_2$  where  $\mu = 1$ .

- (d) Find the shortest distance from the point  $B$  to the line  $l_1$ , giving your answer to 3 significant figures.

(3)

**(Total 11 marks)**

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2. Relative to a fixed origin  $O$ , the point  $A$  has position vector  $\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}$  and the point  $B$  has position vector  $-2\mathbf{i} + 2\mathbf{j} - \mathbf{k}$ . The points  $A$  and  $B$  lie on a straight line  $l$ .

(a) Find  $\overrightarrow{AB}$ .

(2)

(b) Find a vector equation of  $l$ .

(2)

The point  $C$  has position vector  $2\mathbf{i} + p\mathbf{j} - 4\mathbf{k}$  with respect to  $O$ , where  $p$  is a constant.

Given that  $AC$  is perpendicular to  $l$ , find

(c) the value of  $p$ ,

(4)

(d) the distance  $AC$ .

(2)

**(Total 10 marks)**

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3. With respect to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1 : \mathbf{r} = \begin{pmatrix} 4 \\ 28 \\ 4 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ -5 \\ 1 \end{pmatrix}, \quad l_2 : \mathbf{r} = \begin{pmatrix} 5 \\ 3 \\ 1 \end{pmatrix} + \mu \begin{pmatrix} 3 \\ 0 \\ -4 \end{pmatrix}$$

where  $\lambda$  and  $\mu$  are scalar parameters.

The lines  $l_1$  and  $l_2$  intersect at the point  $X$ .

- (a) Find the coordinates of the point  $X$ . (3)

- (b) Find the size of the acute angle between  $l_1$  and  $l_2$ , giving your answer in degrees to 2 decimal places. (3)

The point  $A$  lies on  $l_1$  and has position vector  $\begin{pmatrix} 2 \\ 18 \\ 6 \end{pmatrix}$

- (c) Find the distance  $AX$ , giving your answer as a surd in its simplest form. (2)

The point  $Y$  lies on  $l_2$ . Given that the vector  $\overrightarrow{YA}$  is perpendicular to the line  $l_1$

- (d) find the distance  $YA$ , giving your answer to one decimal place. (2)

The point  $B$  lies on  $l_1$  where  $|\overrightarrow{AX}| = 2|\overrightarrow{AB}|$ .

- (e) Find the two possible position vectors of  $B$ . (3)

(Total 13 marks)

4. With respect to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1: \mathbf{r} = \begin{pmatrix} 6 \\ -3 \\ -2 \end{pmatrix} + \lambda \begin{pmatrix} -1 \\ 2 \\ 3 \end{pmatrix}, \quad l_2: \mathbf{r} = \begin{pmatrix} -5 \\ 15 \\ 3 \end{pmatrix} + \mu \begin{pmatrix} 2 \\ -3 \\ 1 \end{pmatrix},$$

where  $\mu$  and  $\lambda$  are scalar parameters.

- (a) Show that  $l_1$  and  $l_2$  meet and find the position vector of their point of intersection  $A$ .

(6)

- (b) Find, to the nearest  $0.1^\circ$ , the acute angle between  $l_1$  and  $l_2$ .

(3)

The point  $B$  has position vector  $\begin{pmatrix} 5 \\ -1 \\ 1 \end{pmatrix}$ .

- (c) Show that  $B$  lies on  $l_1$ .

(1)

- (d) Find the shortest distance from  $B$  to the line  $l_2$ , giving your answer to 3 significant figures.

(4)

**(Total 14 marks)**

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5. With respect to a fixed origin  $O$ , the lines  $l_1$  and  $l_2$  are given by the equations

$$l_1 : \mathbf{r} = (9\mathbf{i} + 13\mathbf{j} - 3\mathbf{k}) + \lambda (\mathbf{i} + 4\mathbf{j} - 2\mathbf{k})$$

$$l_2 : \mathbf{r} = (2\mathbf{i} - \mathbf{j} + \mathbf{k}) + \mu (2\mathbf{i} + \mathbf{j} + \mathbf{k})$$

where  $\lambda$  and  $\mu$  are scalar parameters.

- (a) Given that  $l_1$  and  $l_2$  meet, find the position vector of their point of intersection.

(5)

- (b) Find the acute angle between  $l_1$  and  $l_2$ , giving your answer in degrees to 1 decimal place.

(3)

Given that the point  $A$  has position vector  $4\mathbf{i} + 16\mathbf{j} - 3\mathbf{k}$  and that the point  $P$  lies on  $l_1$  such that  $AP$  is perpendicular to  $l_1$ ,

- (c) find the exact coordinates of  $P$ .

(6)

**(Total 14 marks)**

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6. Relative to a fixed origin  $O$ , the point  $A$  has position vector  $(2\mathbf{i} - \mathbf{j} + 5\mathbf{k})$ ,

the point  $B$  has position vector  $(5\mathbf{i} + 2\mathbf{j} + 10\mathbf{k})$ ,

and the point  $D$  has position vector  $(-\mathbf{i} + \mathbf{j} + 4\mathbf{k})$ .

The line  $l$  passes through the points  $A$  and  $B$ .

(a) Find the vector  $\overrightarrow{AB}$ .

(2)

(b) Find a vector equation for the line  $l$ .

(2)

(c) Show that the size of the angle  $BAD$  is  $109^\circ$ , to the nearest degree.

(4)

The points  $A$ ,  $B$  and  $D$ , together with a point  $C$ , are the vertices of the parallelogram  $ABCD$ , where  $\overrightarrow{AB} = \overrightarrow{DC}$ .

(d) Find the position vector of  $C$ .

(2)

(e) Find the area of the parallelogram  $ABCD$ , giving your answer to 3 significant figures.

(3)

(f) Find the shortest distance from the point  $D$  to the line  $l$ , giving your answer to 3 significant figures.

(2)

**(Total 15 marks)**

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7. Relative to a fixed origin  $O$ , the point  $A$  has position vector  $\begin{pmatrix} -2 \\ 4 \\ 7 \end{pmatrix}$

and the point  $B$  has position vector  $\begin{pmatrix} -1 \\ 3 \\ 8 \end{pmatrix}$ .

The line  $l_1$  passes through the points  $A$  and  $B$ .

(a) Find the vector  $\overrightarrow{AB}$ .

(2)

(b) Hence find a vector equation for the line  $l_1$ .

(1)

The point  $P$  has position vector  $\begin{pmatrix} 0 \\ 2 \\ 3 \end{pmatrix}$ .

Given that angle  $PBA$  is  $\theta$ ,

(c) show that  $\cos \theta = \frac{1}{3}$ .

(3)

The line  $l_2$  passes through the point  $P$  and is parallel to the line  $l_1$ .

(d) Find a vector equation for the line  $l_2$ .

(2)

The points  $C$  and  $D$  both lie on the line  $l_2$ .

Given that  $AB = PC = DP$  and the  $x$  coordinate of  $C$  is positive,

(e) find the coordinates of  $C$  and the coordinates of  $D$ .

(3)

(f) find the exact area of the trapezium  $ABCD$ , giving your answer as a simplified surd.

(4)

**(Total 15 marks)**

8. With respect to a fixed origin  $O$ , the line  $l$  has equation

$$\mathbf{r} = \begin{pmatrix} 13 \\ 8 \\ 1 \end{pmatrix} + \lambda \begin{pmatrix} 2 \\ 2 \\ -1 \end{pmatrix}, \text{ where } \lambda \text{ is a scalar parameter.}$$

The point  $A$  lies on  $l$  and has coordinates  $(3, -2, 6)$ .

The point  $P$  has position vector  $(-p\mathbf{i} + 2p\mathbf{k})$  relative to  $O$ , where  $p$  is a constant.

Given that vector  $\overrightarrow{PA}$  is perpendicular to  $l$ ,

- (a) find the value of  $p$ .

(4)

Given also that  $B$  is a point on  $l$  such that  $\angle BPA = 45^\circ$ ,

- (b) find the coordinates of the two possible positions of  $B$ .

(5)

(Total 9 marks)

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**TOTAL FOR PAPER: 101 MARKS**